AIAA JOURNAL
Vol. 36, No. 8, August 1998

Multiple-Criterion Method for Determining Structural Damage

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A new multiple-criterion updating method that minimizes the Euclidean norm of the error vector obtained by adding the normalized eigenproblem equation and equation of motion with equal weighting functions is proposed. The method is applied to detecting damage in structures and is tested on an unsymmetrical H-shaped structure. It is found that the multiple-criterion updating method predicts the presence, the position, and the extent of damage. The multiple-criterion method is compared to the frequency-response function method and the modal property-based method by using the coordinate modal assurance criterion and the modal assurance criterion. The multiple-criterion method was found to give better results than the other two methods. This is because it was better able to detect damage on the structure than the modal property method (which failed to detect multiple-damage cases) and gave results that were less noisy, i.e., less updating to undamaged elements, than the frequency-response method.

Nomenclature

$[C]$ = viscous damping matrix
e = Euclidean norm of error
$\{F(\omega)\}$ = system force input vector
$H(\omega)$ = frequency-response function
$[I]$ = identity matrix
i = $\sqrt{-1}$
$[K]$ = stiffness matrix
$[M]$ = mass matrix
$[T]$ = transformation vector
$\{X(\omega)\}$ = response vector
α, β = proportional damping coefficients
$\{\varepsilon\}, \varepsilon$ = error vector, error scalar
$\{\phi\}, [\phi]$ = eigenvector (matrix)
ω = angular frequency
$\{0\}$ = null vector

Superscripts

M = index of the frequency bandwidth of interest
N = number of measured degrees of freedom

Subscripts

i, j, n = index numbers

I. Introduction

DAMAGE to structures may cause failures. The early detection of the presence of damage, its location and extent, may prevent such failures and the detrimental consequences thereof.

Many vibration techniques have been developed to detect damage to structures. D'Ambrogio and Zobel[1] used the frequency-response functions directly to update the finite element model and subsequently detect damage on the structures by minimizing the error in the equation of motion. Baruch[2] used the modal property approach to update finite element models.

In this study, a multiple-criterion method, which simultaneously utilizes both approaches, is proposed. The sum of the errors obtained from the two approaches is minimized iteratively[3] by varying the physical parameters such as the density, cross-sectional area, and the modulus of elasticity of each element, to update the finite element model. Adding the errors from two different approaches will improve the uniqueness of the solution.[4,5]

The finite element model for an undamaged model will be updated by tuning the physical parameters such as modulus of elasticity, density, and Poisson's ratio for each element until the analytical data match the test data. The physical properties of the updated model will be stored for later use. Then damage will be introduced, and the measured data will be used to update the finite element model. The changes in physical properties before and after damage will be used to detect, locate, and quantify the extent of damage. In this paper an unsymmetrical H-shaped structure is studied.

The multiple-criterion method is compared to the frequency-response method and the modal property method by using the modal assurance criterion (MAC)[6] and the coordinate modal assurance criterion (COMAC).[7] The usual MAC is normalized so that a null matrix, instead of the normal unit matrix, would correspond to a perfect correlation between two modes. To simplify the application of the MAC matrix, a scalar factor is introduced as the sum of the squares of elements in the null matrix. Similarly, to simplify the application of the COMAC, a scalar defined as the product of all of the elements in the standard COMAC vector is introduced in this work.

II. Updating Methods

A. Updating Using Measured Frequency Response Functions

In this section a method based on the work done by D'Ambrogio and Zobel[1] is briefly developed. The equation of motion may be written in the frequency domain as follows:

$$(-\omega^2[M] + i\omega[C] + [K])\{X(\omega)\} - \{F(\omega)\} = \{0\} \qquad (1)$$

Assuming, for the purpose of this study, that damping is low, the damping matrix may be assumed to be proportional. Equation (1) may, therefore, be rewritten as

$$(-\omega^2[M] + i\omega(\alpha[M] + \beta[K]) + [K])\{X(\omega)\} - \{F(\omega)\} = \{0\} \qquad (2)$$

where $\{X(\omega)\}$ and $\{F(\omega)\}$ are measured quantities. Generally, frequency-response functions are measured instead of displacement and force individually. Assuming white noise of unit magnitude at all frequencies, the displacement may be replaced by the measured frequency-response functions.

The frequency-response functions are measured at selected degrees of freedom. For this reason the degrees of freedom of measured coordinates are less than that of the finite element model. Consequently, the mass and stiffness matrices in Eq. (2) have to be reduced. In this study the reduction technique chosen is the improved reduced system (IRS).[8] The IRS is an improvement of the Guyan static reduction.[9]

In the Guyan static reduction method, the displacement and force vectors $\{X\}$ and $\{F\}$, and the mass and stiffness matrices in Eq. (2)

Received Nov. 8, 1997; revision received April 15, 1998; accepted for publication April 27, 1998.

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are partitioned into master (measured) and slave (unmeasured) coordinates. If the inertia terms are neglected, the partitioned equation of motion can be used to eliminate the slave coordinates. From this, a transformation matrix is obtained as follows:

$$[\boldsymbol{T}_R] = \begin{bmatrix} [I] \\ -[K]_{ss}^{-1}[K_{sm}] \end{bmatrix} \tag{3}$$

The transformation matrix in Eq. (3) is obtained with the assumption that there is no damage at unmeasured coordinates. This transformation matrix can then be used to obtain the reduced mass and stiffness matrices as follows:

$$[M_R] \approx [\boldsymbol{T}_R]^T[M][\boldsymbol{T}_R], \qquad [K_R] \approx [\boldsymbol{T}_R]^T[K][\boldsymbol{T}_R] \tag{4}$$

The transformation in Eq. (3) can be used in conjunction with the reduced mass and stiffness matrices in Eq. (4), as well as the $[S]$ matrix, to obtain a new transformation equation as follows:

$$[\boldsymbol{T}_{RR}] = [\boldsymbol{T}_R] + [S][M][\boldsymbol{T}_R][M_R]^{-1}[K_R] \tag{5}$$

where

$$[S] = \begin{bmatrix} [0][0] \\ [0][K_{ss}]^{-1} \end{bmatrix} \tag{6}$$

from which new reduced mass and stiffness matrices are obtained as follows:

$$[M_{RR}] \approx [\boldsymbol{T}_{RR}]^T[M][\boldsymbol{T}_{RR}], \qquad [K_{RR}] \approx [\boldsymbol{T}_{RR}]^T[K][\boldsymbol{T}_{RR}] \tag{7}$$

If Eqs. (7) are substituted in Eq. (2), the following expression is obtained:

$$(-\omega^2[M_{RR}] + i\omega[C_{RR}] + [K_{RR}])\{\boldsymbol{X}_m(\omega)\} - \{\boldsymbol{F}_m(\omega)\} = \{\varepsilon\} \tag{8}$$

The reduced damping matrix $[C_{RR}]$ is an idealization. Because the damping is low, it is assumed that damping is proportional to the reduced mass and stiffness matrices.

Because of the cumbersome nature of investigating the elements of the error vector, the Euclidean norm, which is the square root of the sum of the squares of the error vector elements, may be used. The Euclidean norm of this error vector is defined as follows:

$$e = \left(\sum_{j=1}^{M} \varepsilon^2(\omega_j)\right)^{\frac{1}{2}} \tag{9}$$

In the light of a nonzero error vector, the design variables are varied until e is minimized. These design variables may include the cross-sectional area, density, Poisson ratio, and modulus of elasticity of each element.

B. Updating Using the Modal Property Method

The eigenproblem may be written as follows:

$$\left(-\omega_j^2[M] + [K]\right)\{\phi_j\} = \{\boldsymbol{0}\} \tag{10}$$

where ω_j and ϕ_j are the undamped natural frequency and mode shape for mode j, respectively. Equation (10) can be premultiplied by the transpose of the mode shape vector, and the resulting equation is

$$\{\phi_j\}^T\left(-\omega_j^2[M] + [K]\right)\{\phi_j\} = 0 \tag{11}$$

As in the preceding section, the mass and stiffness matrices may be reduced by using the IRS method. The mass and stiffness matrices in Eq. (11) may be substituted by Eqs. (7) and (11), respectively, to obtain

$$\varepsilon_j = \omega_j^2\{\phi_j\}^T[M_{RR}]\{\phi_j\} - \{\phi_j\}^T[K_{RR}]\{\phi_j\} \tag{12}$$

If N modes are extracted, then there will be N error coefficients. As in the preceding section the Euclidean norm of all the ε obtained [see Eq. (9)] may be used to obtain e. The design variables may be similarly varied until e is minimized.

C. Updating Using the Multiple-Criterion Method

The methods presented in the preceding sections have been successfully used to detect damage on the structure. The problem, however, is that the modal property-based method tends to reproduce the measured modal properties whereas the frequency-response function method tends to reproduce the measured frequency-response functions. A method that has a higher probability of reproducing both the measured frequency-response functions and modal properties enhances the probability of obtaining a unique solution. A new multiple-criterion method, which uses both the measured frequency-response functions and modal properties simultaneously, is developed and tested.

If Eqs. (8) and (12) are nondimensionalized and added together with equal weighting functions, the following equation is obtained:

$$\varepsilon = \frac{\sum_{j=1}^{M} |[B_i(\omega_j)]\{\boldsymbol{X}(\omega_j)\} - \{\boldsymbol{F}(\omega_j)\}|}{\sum_{j=1}^{M} |[B_0(\omega_j)]\{\boldsymbol{X}(\omega_j)\} - \{\boldsymbol{F}(\omega_j)\}|} + \frac{\sum_{j=1}^{N}\left[\{\phi_j\}^T\left(\omega_j^2[M_i] - [K_i]\right)\{\phi_j\}\right]}{\sum_{j=1}^{N}\left[\{\phi_j\}^T\left(\omega_j^2[M_0] - [K_0]\right)\{\phi_j\}\right]} \tag{13}$$

where

$$[B_i(\omega)] = -\omega^2[M_{RRi}] + i\omega[C_{RRi}] + [K_{RRi}] \tag{14}$$

where M is the number of frequency lines measured, N is the number of modes measured, i is the subscript for the configuration of design variables, and the subscript 0 indicates the parameters at the initial design variables.

The choice of equal weighting functions was taken because it was assumed that the measured frequency-response function and extracted modal properties have the same level of accuracy. The other assumption that was made, which may influence the choice of weighting functions, was the assumption that the rate of convergence is the same for the two approaches individually.

As in the preceding section, the Euclidean norm of all the ε obtained [see Eq. (9)] may be used to obtain e. The design variables may be varied until e is minimized.

An unsymmetrical H-shaped structure is considered (see Fig. 1). The structure is modeled using the Structural Dynamics Toolbox. This toolbox uses Euler–Bernoulli beam elements,[10] which runs in a MatLab environment.[11] The optimization problem is solved using the Optimization Toolbox,[12] which implements sequential quadratic programming.

The problem of identifying parameters with errors given a set of response functions is ill defined and is not unique. This is because of the errors in the measured data, that is, not all modes are identified and not all degrees of freedom are measured. For this reason, the problem has many local minimum points. To deal with this problem, multiple starting points are used during the optimization.

III. Correlation Criteria

A. MAC

The MAC compares any two mode shapes and is defined by the following equation[6]:

$$\mathrm{MAC}_{jr} = \left\{\sum_{r=1}^{L}\left({}^j\phi_{ar}\,{}^j\phi_{mr}^*\right)\right\}^2 \Bigg/ \sum_{r=1}^{L}\left({}^j\phi_{ar}\right)^2 \sum_{r=1}^{L}\left({}^j\phi_{mr}^*\right)^2 \tag{15}$$

The MAC is a measure of the least squares deviation of the points from the straight-line correlation between two modes.

A value close to 1 suggests that the two mode shapes are well correlated, whereas a value close to 0 indicates that the mode shapes are not correlated. If the mode shape matrix is used in Eq. (15), then the MAC becomes an identity matrix. To simplify comparison of sets of mode shapes originating from different sources, a single-value parameter representative of the MAC matrix is introduced. For this purpose, the MAC matrix is first normalized so that the ideal case would correspond to a null matrix:

$$[\mathrm{MAC}_0] = [I] - [\mathrm{MAC}] \tag{16}$$

where $[I]$ is the identity matrix.

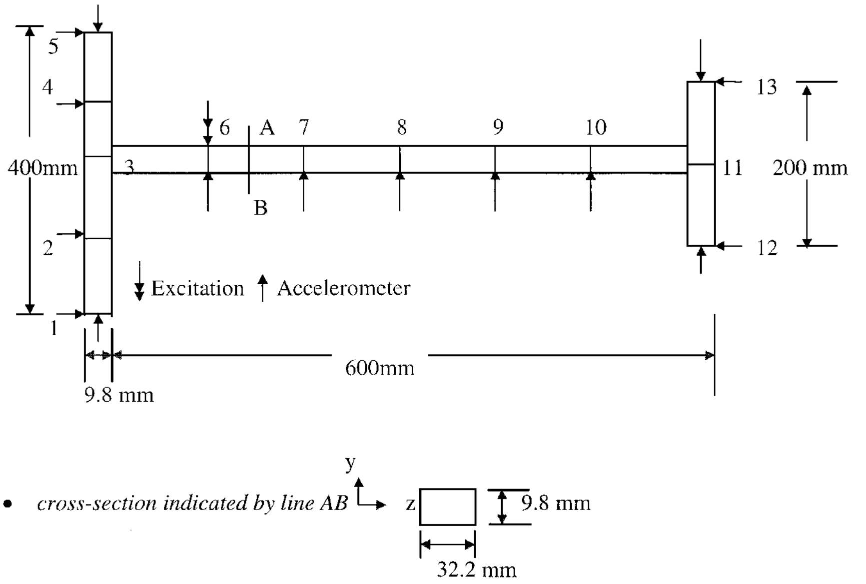

Fig. 1 Irregular H-shaped structure.

If the modal vectors that are being analyzed are perfectly correlated, then the matrix $[\mathrm{MAC}_0]$ will have zero entries. A factor that is the sum of the squares of the elements of $[\mathrm{MAC}_0]$ may be defined as follows:

$$\mathrm{MAC}_{0S} = \sum_{j=1}^{J}\sum_{i=1}^{I}\mathrm{MAC}_{0ij}^2 \tag{17}$$

where I and J are the numbers of rows and columns in the MAC matrix.

If the scalar MAC_{0S} is equal to zero, then the two mode shape matrices are well correlated. One shortcoming with this method is that it does not discriminate between random scatter responsible for the deviations or systematic deviations. The main causes of less than perfect MAC results are nonlinearity in the test structure, noise on the measured data, and poor modal analysis of the measured data.[13] The other shortcoming of the MAC is that it has a problem with comparing the modes that are close in frequency.

B. COMAC

The COMAC method is based on the same principle as the MAC and is essentially an indication of the correlation between the measured and the computed mode shapes for a given common coordinate. The COMAC for coordinate j is given by[7]

$$\mathrm{COMAC}(j) = \left\{\sum_{r=1}^{N}\left|\left({}^{j}\phi_{ar}\right)\left({}^{j}\phi_{mr}^{*}\right)\right|\right\}^2 \Bigg/ \sum_{r=1}^{N}\left({}^{j}\phi_{ar}\right)^2\sum_{r=1}^{N}\left({}^{j}\phi_{mr}^{*}\right)^2 \tag{18}$$

where N is the total number of well-correlated modes as indicated by the MAC. A value close to 1 suggests good correlation. If the mode shape matrices are used, then the COMAC becomes a vector. For a perfect coordinate correlation, the entries of the COMAC vector are all equal to 1.

Unlike the MAC, the COMAC does not have any difficulty comparing modes that are close in frequency or that are measured at insufficient transducer locations. The product of the elements of the entries of the COMAC vector may be defined as follows:

$$\mathrm{COMAC}_s = \prod_{j=1}^{L}\mathrm{COMAC}(j) \tag{19}$$

where L is the number of measured degrees of freedom.

IV. Examples

An unsymmetrical H-shaped (see Fig. 1) aluminium structure was used. The structure was suspended using elastic rubber bands. The structure was excited using an electromagnetic shaker, and the response was measured using an accelerometer.

The structure was divided into 12 elements. It was excited at node 6, and an accelerometer was placed at 15 locations. The structure was tested freely suspended, and a set of 15 frequency-response functions (FRF) was obtained and used for updating. A roving accelerometer was used in the testing. The mass of the accelerometer was found to be negligible compared to the mass of the structure.

The nature of damage introduced was a saw-cut that went about $\frac{1}{4}$ through the cross section of the member. Three damage cases were considered. In the first, damage was introduced at element 3. In the second, damage was introduced at elements 3 and 4, and in the third, damage was introduced at elements 3–5. For undamaged case and each of the damaged cases, a set of 15 FRF was measured and used to extract modal properties.

The inertance was measured and converted into the receptance by dividing the measured data by the negative of the square of the frequency at each frequency point. Damage was introduced on one side of the structure to destroy the symmetry of the structure. This enhanced the ability of the method to detect damage.

The number of measured coordinates was 15, and the number of unmeasured coordinates was 24. The model was reduced from 39 degrees of freedom to 15 degrees of freedom. Damage was introduced at both the partitions.

V. Results

A. Case 1: Before Damage Is Introduced

The number of modes identified is 5. The initial finite element model gave natural frequencies that were on average about 4 Hz from the measured ones. The initial finite element model was refined, and the resulting model was on average within 1 Hz of the measured results. These results are shown in Table 1.

When the three updating procedures are applied, three updated finite element models are obtained. Each updated finite element model predicts a set of mode shapes. These mode shapes are compared to the corresponding experimental mode shapes using the COMAC and the MAC scalars. The results obtained are shown in Table 2.

The results show that on average the modal property-based method gives the best co-ordinated modes, followed by the multiple-criterion method, and then the FRF method. The MAC also shows the same trend.

B. Case 2: Damage at Element 3

The same procedure as in the preceding section is followed. The results obtained are shown in Table 3. The COMAC results show that the modal property method gives the best results, followed by the multiple-criterion method, and then the FRF method. The MAC also shows the same trend.

The FRF method in Fig. 2 shows that the location of damage is at element 3 and has a magnitude of about 100×10^8 Pa. The modal

Table 1 Comparison of natural frequencies before damage

Mode	Experimental natural frequency, Hz	Initial natural frequency, Hz	Updated natural frequency, Hz, FRF	Updated natural frequency, Hz, modal	Updated natural frequency, Hz, multiple
1	55.3	58.2	55.3	55.4	55.3
2	125.3	128.0	125.0	125.0	125.3
3	225.2	224.5	226.7	224.7	125.3
4	259.7	263.4	258.7	258.7	225.4
5	446.0	450.9	444.4	444.6	445.5

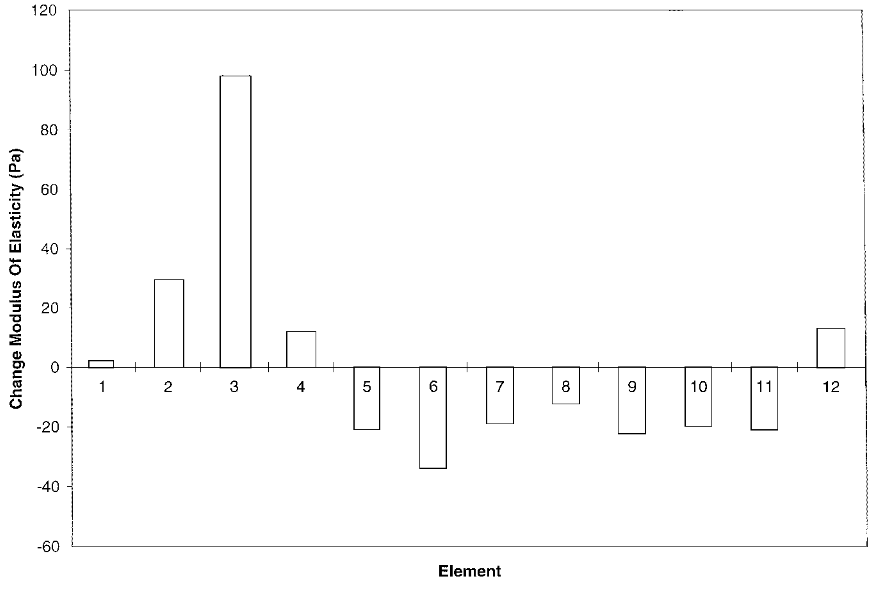

Fig. 2 Damage case 1 using FRFs.

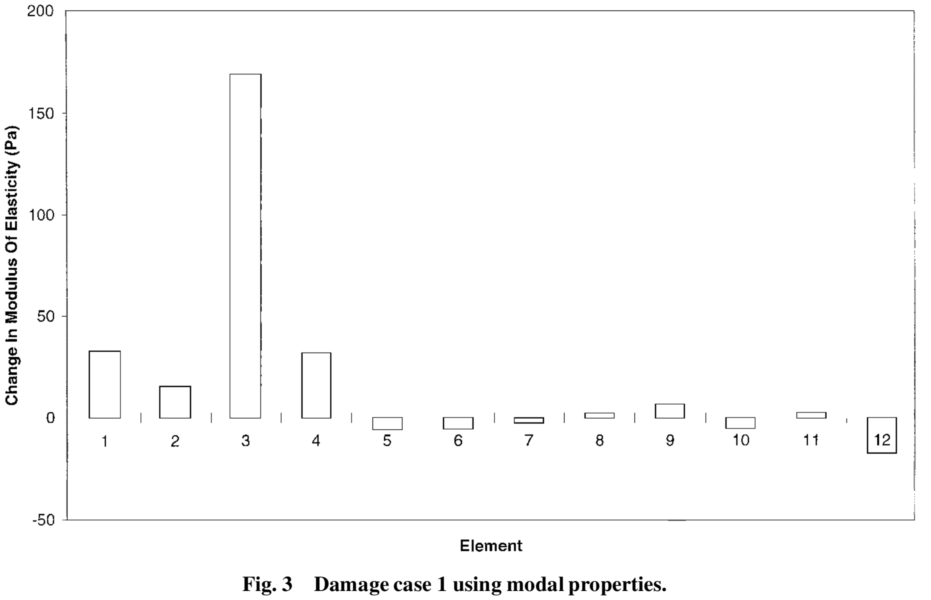

Fig. 3 Damage case 1 using modal properties.

property method in Fig. 3 shows that the location of damage is at element 3 and has a magnitude of about 150×10^8 Pa. The multiple-criterion method in Fig. 4 indicates that the location of damage at element 3 and has a magnitude of about 140×10^8 Pa. It should be noted that the frequency-response method (see Fig. 2) shows more updating to undamaged elements than the other two methods (see Figs. 3 and 4). This shows that the FRF method is more sensitive to noise in the measurement than the multiple-criterion method.

C. Case 3: Damage at Element 3 and 4

The comparison results obtained in this case are shown in Table 4. The results show the same trend as in preceding section.

The FRF method (in Fig. 5) shows that the locations of damage are at elements 3 and 4 and have magnitudes of about 200×10^8 and 225×10^8 Pa, respectively. The modal property method (in Fig. 6) shows that the locations of damage are at element 3 and 4 and have magnitudes of about 100×10^8 Pa. The multiple-criterion method

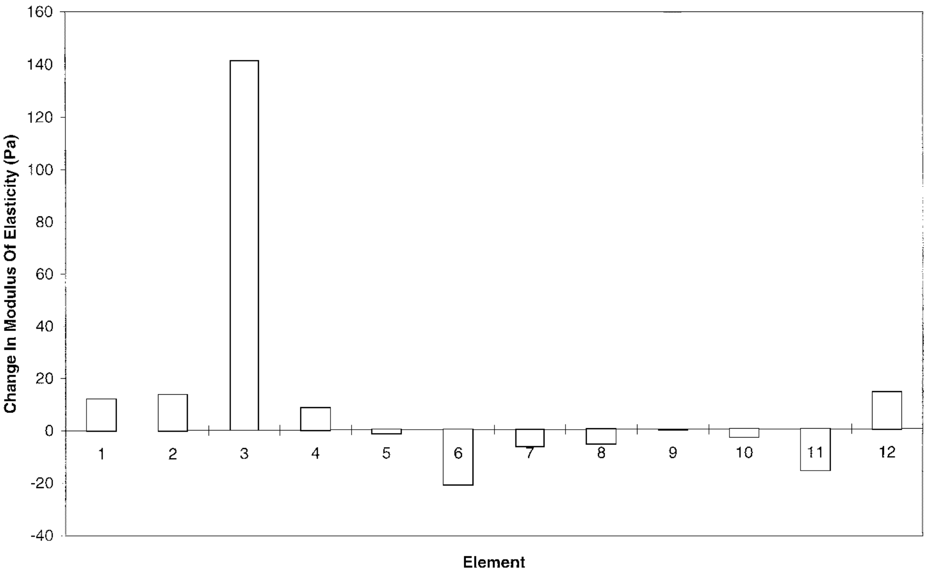

Fig. 4 Damage case 1 using multiple-criterion method.

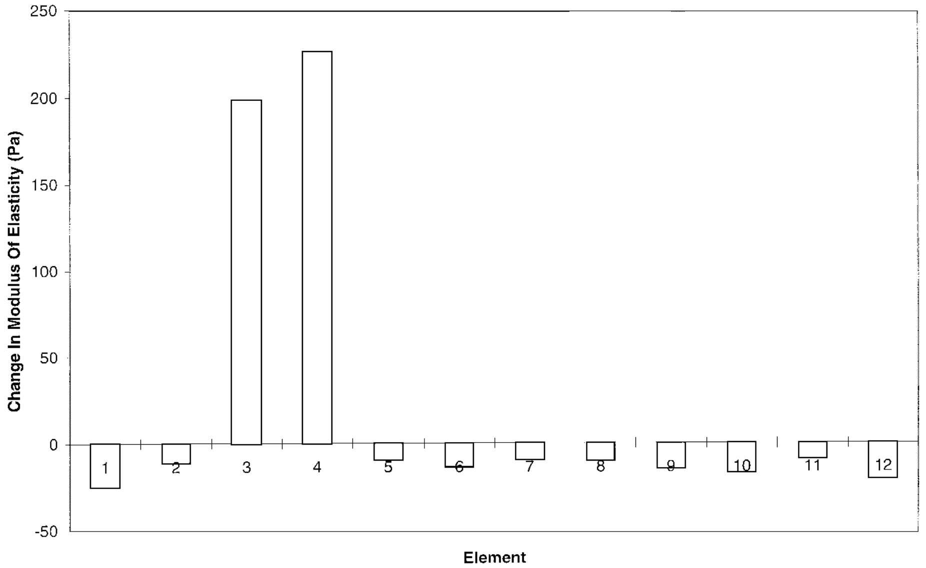

Fig. 5 Damage case 2 using FRFs.

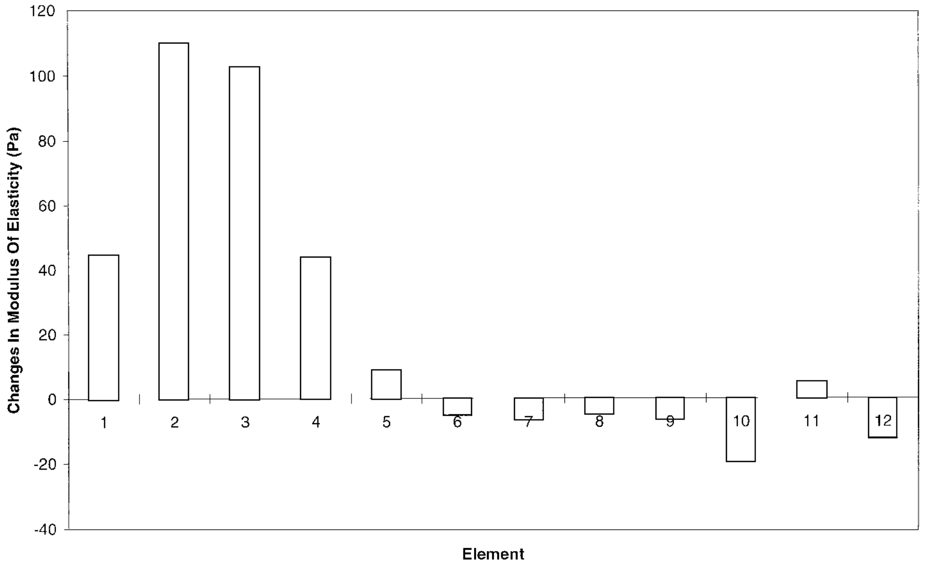

Fig. 6 Damage case 2 using modal properties.

(in Fig. 7) indicates that the locations of damage are at element 3 and 4 and with magnitudes of 140×10^8 and 100×10^8 Pa, respectively. The modal property-method (see Fig. 6) shows significant updating to the first and fourth elements even though those elements are undamaged. The multiple-criterion method and the FRF approach were, therefore, better able to locate damage.

D. Case 4: Damage at Elements 3–5

The comparison results obtained in this case are shown in Table 5. The results show the same trend as in preceding section.

The FRF method (in Fig. 8) shows that the locations of damage are at elements 3, 4, and 5 and have magnitudes of about 200×10^8, 225×10^8, and 150×10^8 Pa, respectively. The modal property method (in Fig. 9) is not able to locate damage. However, because significant updating is performed indicates that the method was able to detect the presence of damage. The multiple-criterion method (in Fig. 10) indicates the locations of damage are at elements 3, 4, and 5 and with magnitudes of about 200×10^8, 75×10^8, and 125×10^8 Pa, respectively.

VI. Discussion

The graphs in Figs. 5 and 9 show that the modal property approach fails to detect the location of damage for the multiple-damage case. This is because the modal properties have less information than the FRFs. The graphs in Figs. 2, 5, and 8 show that the FRF approach tends to update more parameters without damage than the multiple-criterion approach. This is because the FRFs has more noise than the modal properties. The multiple criterion approach is able to identify all damage cases.

The multiple-criterion method experienced several problems. One of these problems was that it requires many iterations and is, therefore, computationally expensive. It was found that by scaling the equations of motion in the multiple-criterion method, however, that the number of iterations may be reduced. In this study, the equation of motion was nondimensionalized by dividing the FRF-based method formulation by the value of the error it gives at the initial design variables. The same was done to the modal property method formulation.

Furthermore, the nature of the damage introduced to the structure was a saw cut. In real structures, the main causes of damage include fatigue. In fatigue damage, the presence of damage tends to increase the level of damping on the structure. Because of this, the proportional damping assumption may no longer hold and, consequently, complicated modeling of damping may be required and may compromise the effectiveness of the multiple-criterion method.

The other issue pertaining to damage involves the location of damage. Damage was generally introduced only on one-half of the structure. This was done purposefully to destroy the symmetry of the structure, thereby increasing the probability of the updating method to detect the presence, location, and the extent of damage. However, in reality the presence of damage might not necessarily destroy the symmetry of the structure. Because of this, the proposed updating method needs to be investigated for randomly introduced damage.

In this study, it was hoped that the extent in which physical parameters would change as a result of damage would correlate well to the extent of damage that occurred. It was discovered, however, that the issue of relating changes in physical parameters to the extent of damage was difficult because it requires highly accurate experimental data, which could not be achieved in this study. It is postulated

Table 2 Comparison of measured and analytical mode shapes before damage

Method	$COMAC_s$	MAC_s
FRF	0.972	0.0658
Modal property	0.987	0.0653
Multiple criterion	0.977	0.0657

Table 3 Comparison of measured and analytical mode shapes for damage case 1

Method	$COMAC_s$	MAC_s
FRF	0.979	0.0724
Modal property	0.992	0.0721
Multiple criterion	0.989	0.0723

Table 4 Comparison of measured and analytical mode shapes for damage case 2

Method	$COMAC_s$	MAC_s
FRF	0.9022	0.0166
Modal property	0.9232	0.0119
Multiple criterion	0.9191	0.0161

Table 5 Comparison of measured and analytical mode shapes for damage case 3

Method	$COMAC_s$	MAC_s
FRF	0.9114	0.0309
Modal property	0.9890	0.0304
Multiple criterion	0.9373	0.0299

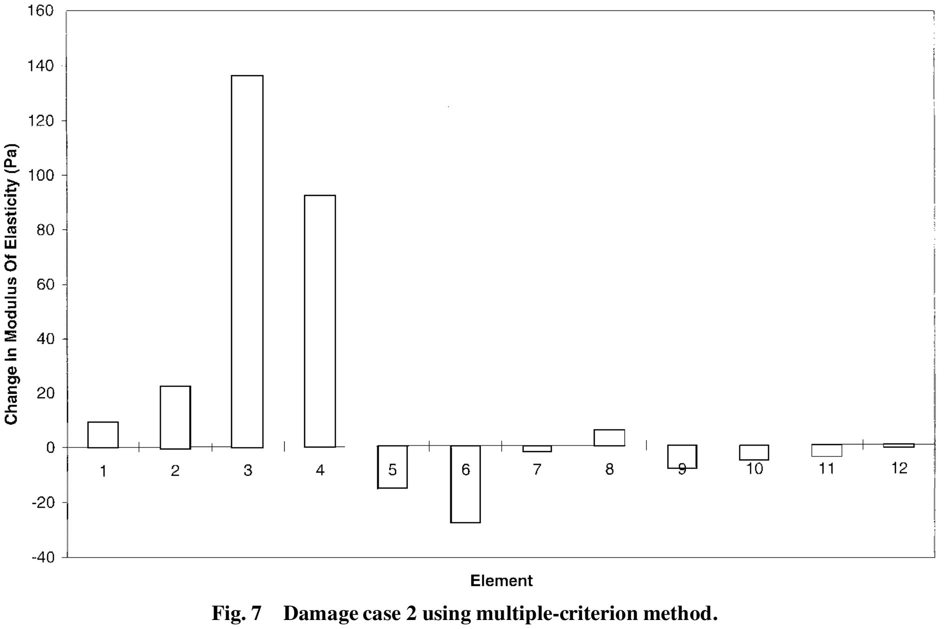

Fig. 7 Damage case 2 using multiple-criterion method.

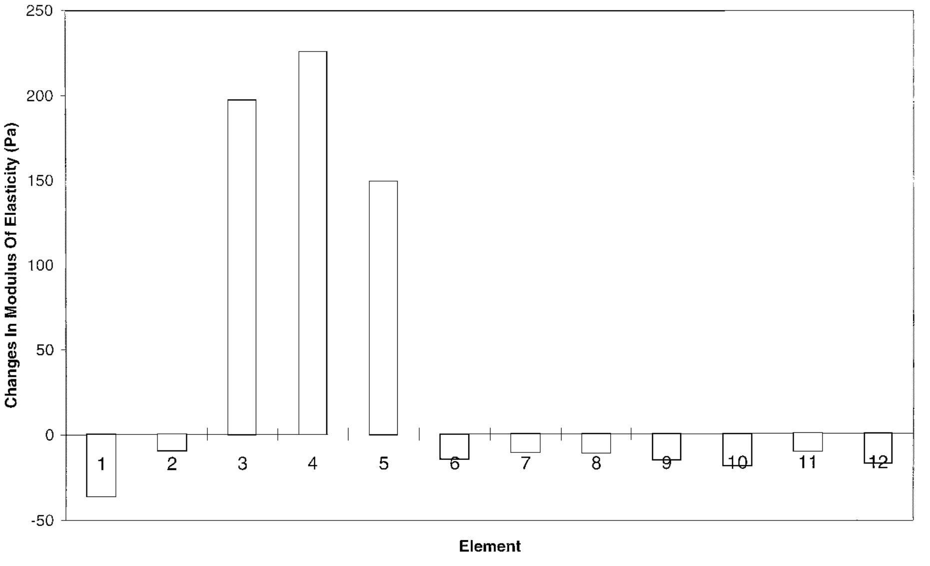

Fig. 8 Damage case 3 using FRFs.

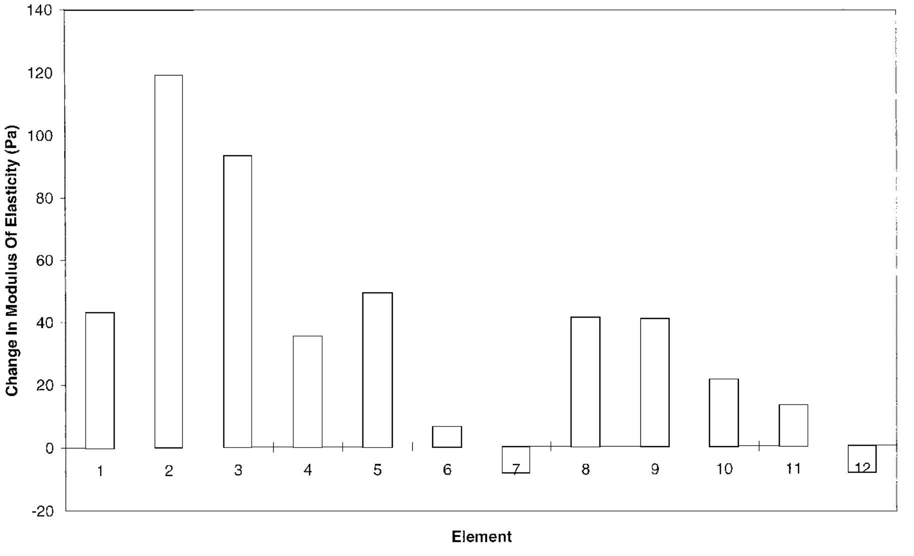

Fig. 9 Damage case 3 using modal properties.

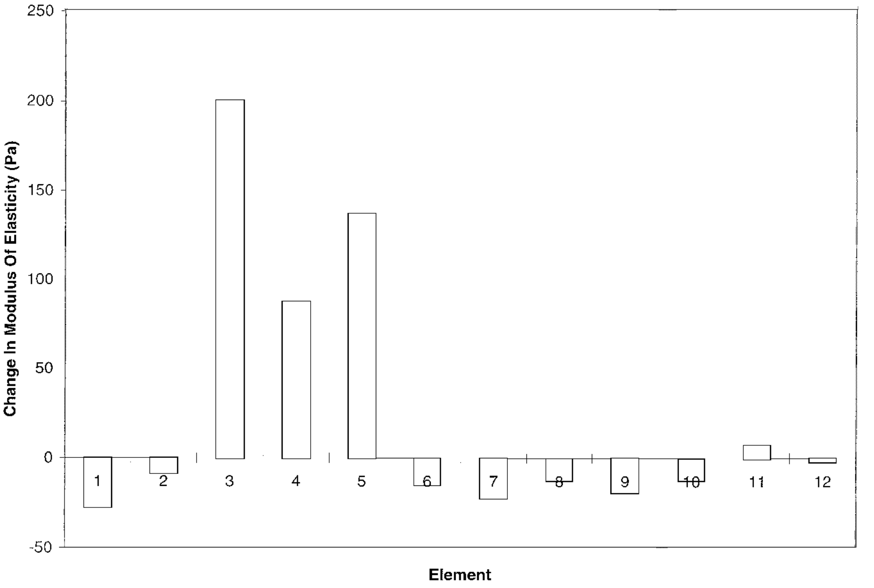

Fig. 10 Damage case 3 using multiple-criterion method.

that the relationship between the changes in modulus of elasticity and the amount of damage that has occurred is a function of the material property of the structure, and further investigation is required.

VII. Conclusion

A new multiple-criterion method was developed and tested using an unsymmetrical H-shaped structure. Because this method is a multiple-criterion method, other conditions may be added to the multiple-criterion equation so that the probability of obtaining a unique solution is increased. It is concluded that the multiple-criterion method gives better results than the modal property method and the FRF method. This is because it was better able to detect damage on the structure than the modal property method (which failed to detect multiple damage cases) and gave results that were less noisy, i.e., less updating to undamaged elements, than the frequency-response method.

Acknowledgments

Financial support was received from the AECI, Ltd., and the Foundation for Research and Development. The authors would like to thank the University of Pretoria for supplying all of the equipment for this research. The authors would like to thank the following people: Shaun MacWherter, Andre Engelbreght, and Nielen Stander.

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